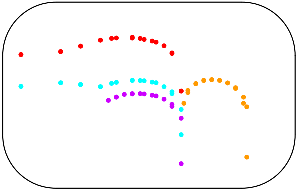


Synchronization of an Opto Mechanical Oscillator

Maria Martin

February 7, 2008

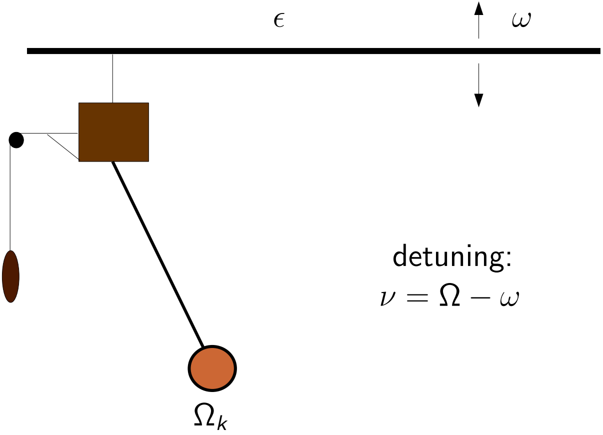
Outline

Synchronization

What is a self-sustained oscillator?

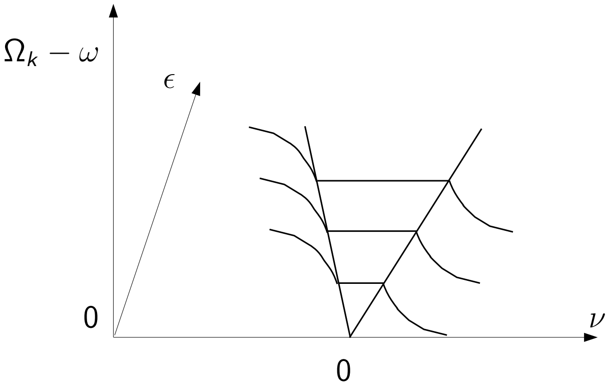
… an autonomous dissipative system with a stable oscillation on a limit cycle.

e.g.: a pendulum clock



detuning:
$\nu = \Omega - \omega$

Arnold-tounge



Synchronizability:

Width of synchronization-plateau depends on coupling strength ϵ (strictly proportional only in used approximation).

The Opto-mechanical Oscillator

What is the OMO ?

Comparison of a pendulum clock with an **O**pto **M**echanical **O**scillator:

Systems:	Pendulum Clock	OMO

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what oscillates?	pendulum	mechanical element
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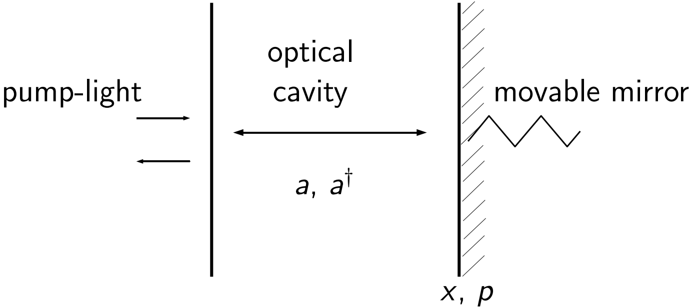
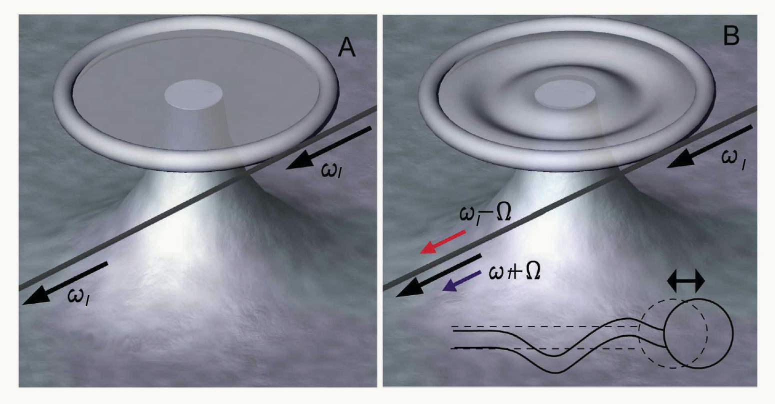
several possible implementations...

Implementation à la Vitali



Implementation à la Vahala



Hamiltonian

$$H = H_{\text{mech}} + H_{\text{cav}} + H_{\text{pump}} + H_{\text{ia}} + H_{\text{baths}}$$

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Equations of motion

$$\begin{aligned}\ddot{x} &= -x - \gamma\dot{x} + ga^\dagger a + \Gamma_x \\ \dot{a} &= -\mathrm{i}\underbrace{(\omega_c - \omega_l)}_{=:\Delta} a - \kappa a + \mathrm{i}gxa + \kappa + \Gamma_a\end{aligned}$$

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$g a^\dagger a$:

intensity-dependent force

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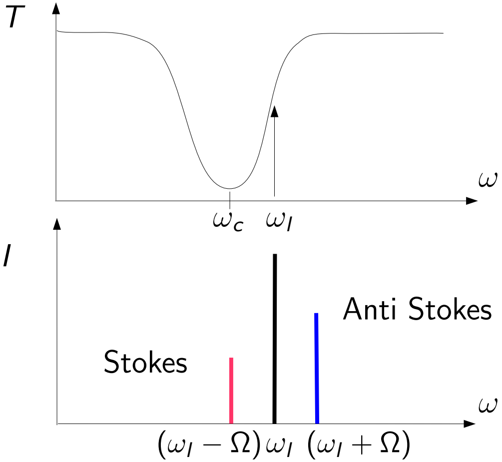
$ga^\dagger a$:

intensity-dependent force

$\mathrm{i}gxa$:

length-dependent frequency-modulation

Properties of the OMO



Limit Cycles of the OMO

A Limit Cycle

$$\boxed{x(t) = \bar{x} + B\cos(\Omega t + \varphi_0)}$$

... for an analytical theory:

derive $a^\dagger a(t)$ from equations of motion for $\dot{a}$, $\dot{a}^\dagger$ and $\ddot{x}$ under limit-cycle-condition and compare results:

$\longrightarrow$ 3 equations (with physical meaning of $\langle force \rangle$,$\langle energy \rangle$ and $\langle power \rangle$)

$\longrightarrow$ 3 unknown parameters $\bar{x}$, B and Ω.

Analytical theory

- Solution for Ω very stable under variation of system parameters

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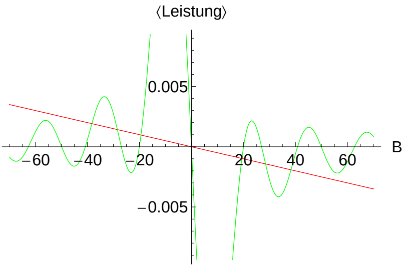

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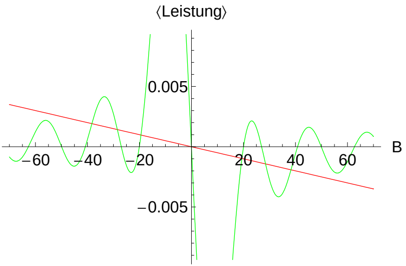

Limit cycles:

- green: $\langle P_{\text{fric}} \rangle$ and red: $\langle P_{\text{rad}} \rangle$

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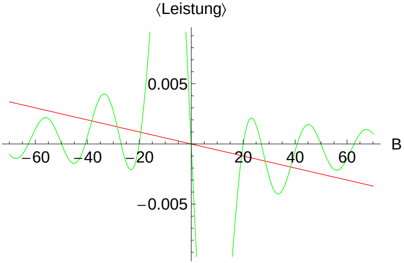
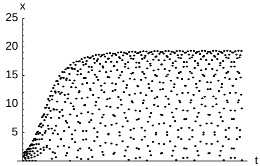


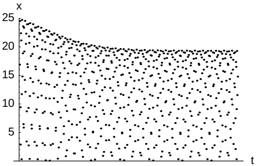
Limit cycles:

- green: $\langle P_{\text{fric}} \rangle$ and red: $\langle P_{\text{rad}} \rangle$
- only stable solutions yield limit cycles

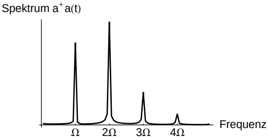
Simulation

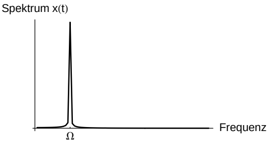
Developement towards a limit cycle:





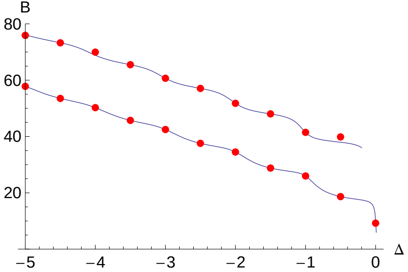
Higher harmonics in light-intensity:





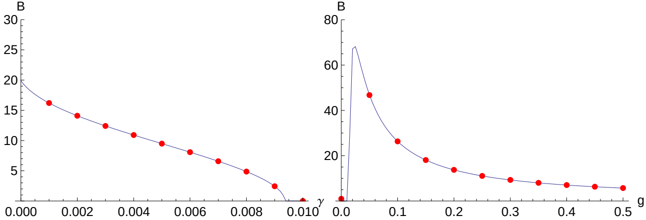
Thresholds for limit cycles

Threshold in optical detuning Δ:

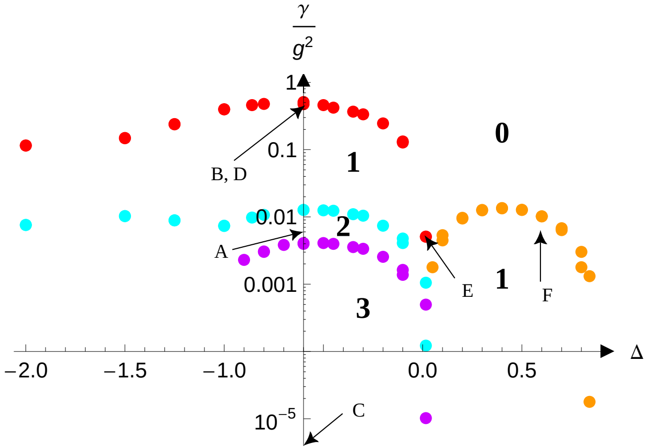


Thresholds for limit cycles

Thresholds in mechanical damping γ and coupling g:



Phase diagram for limit cycles



Critical parameters for number of limit cycles are:

optical detuning Δ and relation γ/g^2

Synchronization of the OMO

Techniques

- Define a Phase $\Phi(t) = \Omega t + \varphi_0$ on the limit cycle $x(t) = \bar{x} + B\cos\Phi$.

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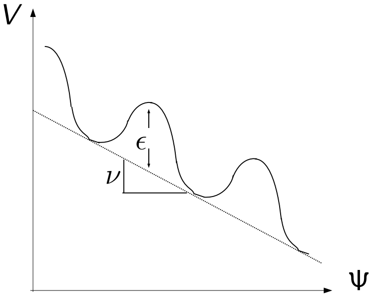
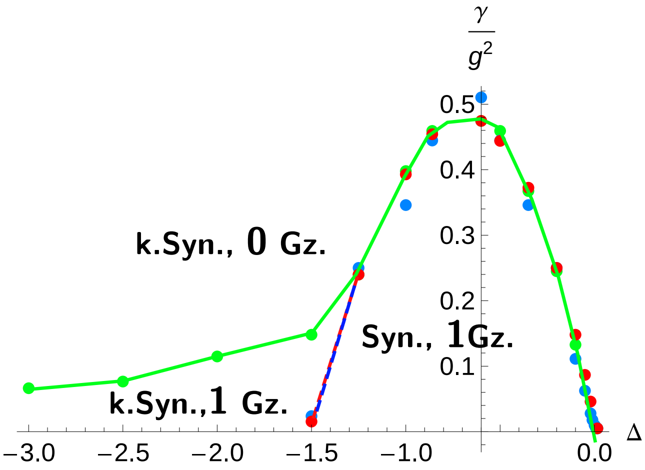
- Define a potential $V(\Psi) := \nu\Psi - \epsilon \int^{\Psi} q(x)dx$
- $\boxed{\textit{Synchronization in local minima}!}$



Phase diagram for synchronization and limit cycles



Synchronization in the presence of noise

- Find dynamics for Ψ: $\dot{\Psi} = -\nu + \epsilon q(\Psi) + \Gamma_{\Psi}$ (See Pikovsky et al.)

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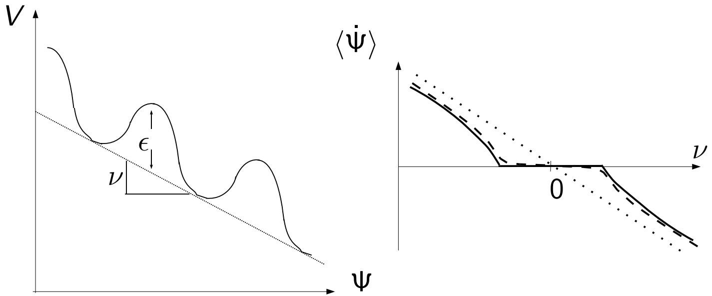
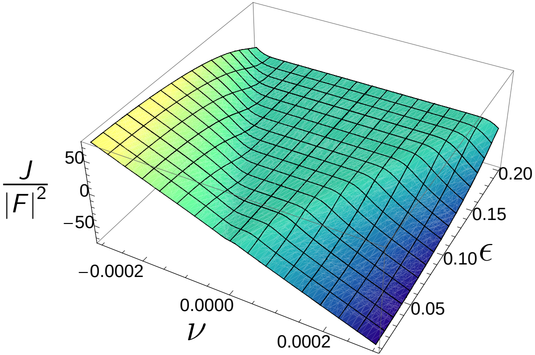
Synchronization in the presence of noise

- Find dynamics for Ψ: $\dot{\Psi} = -\nu + \epsilon q(\Psi) + \Gamma_\Psi$ (See Pikovsky et al.)
- $\boxed{\textit{Synchronization} \Longleftrightarrow \langle\dot{\Psi}\rangle \approx 0}$
- Find a Fokker-Planck-Equation for Ψ to determine $\langle\dot{\Psi}\rangle$



Arnold-tounge for OMO: quasi-deterministic case



Effects of noise:

Only negligible effects on Synchronizability of OMO!

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Phase-noise measurement in a cavity with a movable mirror undergoing quantum brownian motion.
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