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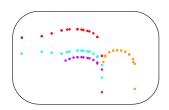
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Maria Martin

February 7, 2008

Maria Martin

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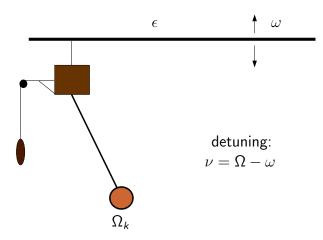
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What is a self-sustained oscillator?

... an autonomous dissipative system with a stable oscillation on a limit cycle.

e.g.: a pendulum clock



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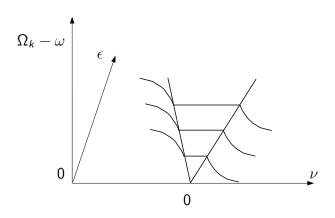
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Synchronizability:

Width of synchronization-plateau depends on coupling strength ϵ (strictly proportional only in used approximation).

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what oscillates?	pendulum	mechanical element

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what oscillates?	pendulum	mechanical element
why stable?	weight	pump-light

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why stable?	weight	pump-light

several possible implementations...

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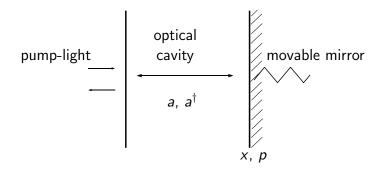
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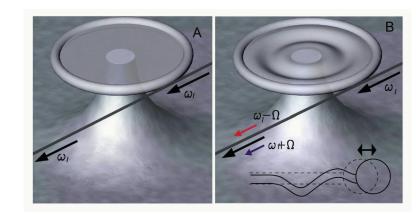
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$$H = H_{\mathsf{mech}} + H_{\mathsf{cav}} + H_{\mathsf{pump}} + H_{\mathsf{ia}} + H_{\mathsf{baths}}$$

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$$H = H_{\text{mech}} + H_{\text{cav}} + H_{\text{pump}} + H_{\text{ia}} + H_{\text{baths}}$$

$$H_{\text{mech}} = \frac{1}{2} \left(p^2 + x^2 \right)$$

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m mech} = rac{1}{2} \left(p^2 + x^2
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 $H_{
m cav} = \omega_c a^\dagger a$

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$$H = H_{\text{mech}} + H_{\text{cav}} + H_{\text{pump}} + H_{\text{ia}} + H_{\text{baths}}$$

$$\begin{array}{lcl} H_{\rm mech} & = & \frac{1}{2} \left(p^2 + x^2 \right) \\ H_{\rm cav} & = & \omega_c a^\dagger a \\ H_{\rm pump} & = & {\rm i} \kappa \left({\rm e}^{-{\rm i} \omega_I t} a^\dagger - {\rm e}^{{\rm i} \omega_I t} a \right) \end{array}$$

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 $H = H_{\mathsf{mech}} + H_{\mathsf{cav}} + H_{\mathsf{pump}} + H_{\mathsf{ia}} + H_{\mathsf{baths}}$

$$H_{\text{mech}} = \frac{1}{2} (p^2 + x^2)$$

$$H_{\text{cav}} = \omega_c a^{\dagger} a$$

$$H_{\text{pump}} = i\kappa \left(e^{-i\omega_l t} a^{\dagger} - e^{i\omega_l t} a \right)$$

$$H_{\text{ia}} = -g \times a^{\dagger} a$$

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Equations of motion

$$\ddot{x} = -x - \gamma \dot{x} + g a^{\dagger} a + \Gamma_x$$

$$\dot{a} = -i\underbrace{(\omega_c - \omega_l)}_{=: \Lambda} a - \kappa a + igxa + \kappa + \Gamma_a$$

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$$\dot{a} = -i\underbrace{(\omega_c - \omega_l)}_{=:\Lambda} a - \kappa a + igxa + \kappa + \Gamma_a$$

ga[†]a :

intensity-dependent force

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$$\dot{a} = -i\underbrace{(\omega_c - \omega_l)}_{=:\Delta} a - \kappa a + igxa + \kappa + \Gamma_a$$

ga[†]a :

intensity-dependent force

i*gxa* :

length-dependent frequency-modulation

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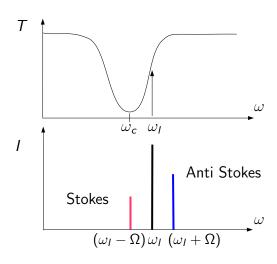
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A Limit Cycle

$$x(t) = \bar{x} + B\cos(\Omega t + \varphi_0)$$

... for an analytical theory:

derive $a^{\dagger}a(t)$ from equations of motion for \dot{a} , \dot{a}^{\dagger} and \ddot{x} under limit-cycle-condition and compare results:

- \longrightarrow 3 equations (with physical meaning of $\langle force \rangle$, $\langle energy \rangle$ and $\langle power \rangle$)
- \longrightarrow 3 unknown parameters \bar{x} , B and Ω .

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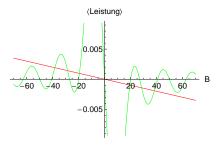
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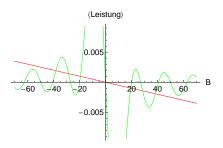
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Limit cycles:

• green: $\langle P_{\mathsf{fric}} \rangle$ and red: $\langle P_{\mathsf{rad}} \rangle$

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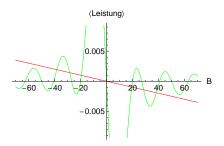
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- Solution for \bar{x} very small in comparison to Amplitude B



Limit cycles:

- green: $\langle P_{\mathsf{fric}} \rangle$ and red: $\langle P_{\mathsf{rad}} \rangle$
- only stable solutions yield limit cycles

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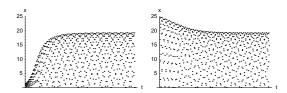
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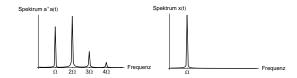
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Developement towards a limit cycle:



Higher harmonics in light-intensity:



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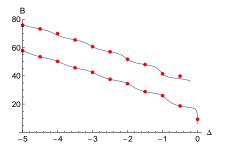
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Thresholds for limit cycles

Threshold in optical detuning Δ :



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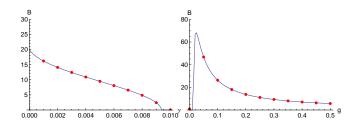
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Thresholds in mechanical damping γ and coupling g:



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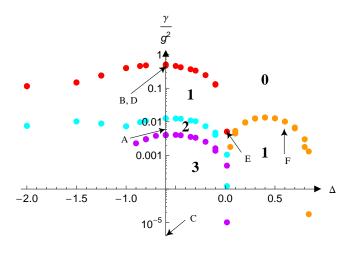
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Critical parameters for number of limit cycles are: optical detuning Δ and relation γ/g^2

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• Define a Phase $\Phi(t) = \Omega t + \varphi_0$ on the limit cycle $x(t) = \bar{x} + B \cos \Phi$.

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- | Synchronization $\iff \dot{\Psi} = 0$

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- Synchronization $\iff \dot{\Psi} = 0$
- Define a potential $V(\Psi) := \nu \Psi \epsilon \int^{\Psi} q(x) dx$

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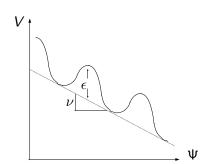
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- Synchronization $\iff \dot{\Psi} = 0$
- Define a potential $V(\Psi) := \nu \Psi \epsilon \int^{\Psi} q(x) dx$
- Synchronization in local minima!



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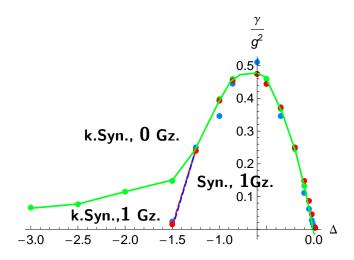
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• Find dynamics for Ψ : $\dot{\Psi} = -\nu + \epsilon q(\Psi) + \Gamma_{\Psi}$ (See Pikovsky et al.)

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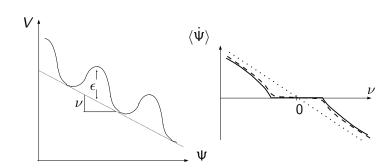
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Synchronization in the presence of noise

- Find dynamics for Ψ : $\dot{\Psi}=-\nu+\epsilon q(\Psi)+\Gamma_{\Psi}$ (See Pikovsky et al.)
- Synchronization $\iff \langle \dot{\Psi} \rangle \approx 0$
- \bullet Find a Fokker-Planck-Equation for Ψ to determine $\langle\dot{\Psi}\rangle$



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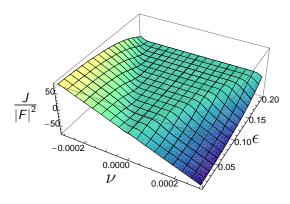
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Effects of noise:
Only negligible effects on Synchronizability of OMO!

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