Einführung in die Quantenoptik II Sommersemester 2008

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Problem set 4 Hand out: 21 May 2008
hand in: 03 June 2008

Problem 4.1 – Exact solution for spontaneous decay (9 points)

Consider a two-level atom (level splitting $\hbar\omega_{\rm A}$) coupled within the rotating-wave approximation to a bosonic environment,

$$H_{\rm int} = \sum_k \left(g_k \sigma^\dagger a_k + g_k^* a_k^\dagger \sigma \right) \tag{4.1}$$

(i) Show that when you start in the state $|\psi(0)\rangle = |{\rm e}, {\rm vac}\rangle$, the system remains in the subspace spanned by $|{\rm e}, {\rm vac}\rangle$ and the states $|{\rm g}, 1_k\rangle$ (atom in ground state and one photon in mode k). (ii) Write down the Schrödinger equation for the probability amplitudes $c_{\rm e}$ and $c_k = \langle {\rm g}, 1_k | \psi(t) \rangle$ of the state $|\psi(t)\rangle$, solve the equation for c_k formally (given the initial condition) and insert the solution into the equation for $c_{\rm e}$. Show that one gets an integro-differential equation of the form

$$ {\rm i}\dot{c}_{\rm e}(t) = \int\limits_0^t {\rm d}t' \, \chi(t - t') c_{\rm e}(t') \tag{4.2}$$

where the "memory function" $\chi(t - t')$ depends on the spectral strength of the environment. (iv) Solve Eq.(4.2) with the help of the Laplace transform. Consider the following examples of spectral functions (notation as in the lecture):

$$S(\omega) = \kappa = {\rm const.}, \qquad S(\omega) = \frac{A\kappa}{(\omega - \omega_{\rm c})^2 + \kappa^2} \tag{4.3}$$

and comment on the results.

Problem 4.2 – Entropy increase and quantum (no)jumps (5 points)

In the lecture, we have seen the interpretation in terms of quantum jumps for the Lindblad master equation. (i) Consider the following change of a two-level density operator over one time step ${\rm d}t$:

$$ {\rm d}\rho = -\frac{\gamma {\rm d}t}{2} \left\{ \sigma^\dagger \sigma, \rho \right\} \tag{4.4}$$

This is the change when "no photon is observed", up to the normalization of the density operator. Restore the normalization (i.e., ${\rm tr}\,\rho = 1$ at all times) and compute the change in entropy of the quantum state. What happens to lowest order in $\mathrm{d}t$? How does the change in entropy depend on the current state ρ? (ii) Now, when the "no-jump" operation is "mixed" with a quantum jump towards the ground state, we have

$$\mathrm{d}\rho = -\frac{\gamma \mathrm{d}t}{2} \left\{ \sigma^\dagger \sigma, \rho \right\} + \gamma \mathrm{d}t \, \sigma \rho \sigma^\dagger \tag{4.5}$$

Enforce the normalization and compute again the change in entropy.

Problem 4.3 – Time-averaged force (6 points)

In the lecture, we have sketched a classical picture for the force acting on a bound system of two charges (opposite sign, distance vector short compared to the wavelength). Work out the details and confirm the expression for the time-averaged force given in the lecture, provided a stationary regime has been reached, i.e., both the fields and the dipole moment of the charges oscillate at a common frequency ω_{L}.