Quanten-Informatik und theoretische Quantenoptik II

Sommersemester 2010

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Problem set 4 Hand out: 07 Jun 2010 hand in: 14 Jun 2010

Problem 4.1 – Lindblad form for spontaneous decay (10 points)

Einstein's model for a two-level atom that exchanges photons with a thermal radiation field can be studied with the atom-field interaction

$$H_{\rm AF} = B^\dagger \sigma + \sigma^\dagger B, \qquad B = \sum_k g_k a_k \tag{4.1}$$

where k labels the modes of the field with annihilation operators a_k and coupling constants g_k. Derive, as we did it in the lecture for the dephasing process, the Lindblad form of the master equation and show that there are two Lindblad operators

$$L_{\rm em} = \sqrt{\gamma(1+\bar{n})}\,\sigma, \qquad L_{\rm abs} = \sqrt{\gamma\bar{n}}\,\sigma \tag{4.2}$$

where $\bar{n}$ is the mean thermal photon number at the resonance frequency $\omega_k = \omega_{\rm A}$ and γ is the spontaneous decay rate ($|{\rm vac}\rangle$ is the vacuum state of the field, $\hbar = 1$)

$$\gamma = \int\limits_{-\infty}^{\infty} {\rm d}\tau\, {\rm e}^{{\rm i}\omega_A \tau} \langle {\rm vac}| B(t+\tau) B^\dagger(t) |{\rm vac}\rangle \tag{4.3}$$

Remark. The time step Δt must be large compared to the optical period $2\pi/\omega_{\rm A}$ for Eq.(4.3) to make sense. Why?

Problem 4.2 – Solution of the Bloch equations (10 points)

The Bloch equations describe the motion of a two-level atom coupled to a coherent laser field (complex Rabi frequency Ω, detuning $\Delta = \omega_{\rm L} - \omega_{\rm A}$)

$$\frac{{\rm d}\langle\sigma\rangle}{{\rm d}t} = ({\rm i}\Delta - \Gamma)\,\langle\sigma\rangle + {\rm i}\Omega\langle\sigma_3\rangle \tag{4.4}$$

$$\frac{{\rm d}\langle\sigma_3\rangle}{{\rm d}t} = {\rm i}\Omega^*\langle\sigma\rangle - {\rm i}\Omega\langle\sigma^\dagger\rangle - \gamma(\langle\sigma_3\rangle + 1) \tag{4.5}$$

This is a linear system of three real differential equations for the components $\{{\rm Re}\langle\sigma\rangle, {\rm Im}\langle\sigma\rangle, \langle\sigma_3\rangle\}$ of the Bloch vector. Write down the solution of these equations for given initial conditions. Show that you get (possibly complex) eigenfrequencies $\omega_{1,2,3}$ that are the roots of the equation

$$0 = {\rm i}\omega^3 + (2\Gamma + \gamma)\omega^2 - {\rm i}(\Delta^2 + |\Omega|^2 + 2\Gamma\gamma + \Gamma^2)\omega - \gamma\Delta^2 \tag{4.6}$$

Conclude that the imaginary parts sum to

$$\mathrm{Im}(\omega_1 + \omega_2 + \omega_3) = -(2\Gamma + \gamma) \tag{4.7}$$

Which condition separates the cases of three purely imaginary eigenfrequencies from one imaginary and two complex ones?

[**5 Bonus points.**] Consider the three limiting cases $\Delta = 0$ or $\Omega = 0$ or $\Gamma \to \infty$ and give a physical interpretation.