Optik an Grenzflächen und Nanostrukturen Sommersemester 2017
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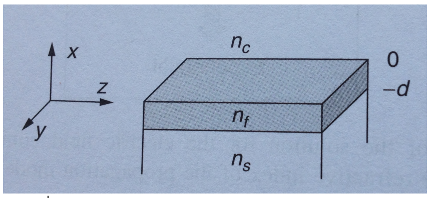
Aufgabe 6.1 – Fiber optics (12 Punkte)

Glass fibres are now a standard tool for communication at high data rates. We explore in this problem a little bit of their physical background.

(1) Light can be guided in a layer (film) with index n_f when the surrounding medium has a lower index. What is the mechanism for this? Sketch a geometrical ray that is bouncing back and forth between the surfaces of the layer.

(2) A light field guided in the film is described by electromagnetic fields whose wave vector along the direction of propagation (the z-axis in Fig. 6.1) is constant and called β. (Optical engineers re-scale it to an 'effective index' $N = \beta c/\omega$.) Why does β not depend on the z-coordinate? Give a lower limit to β, based on the indices given in the table.



c	cladding	$\gamma_\text{c}^2 = \beta^2 - k^2 n_\text{c}^2$	$n_\text{c} = 1.0$
f	film	$\kappa_\text{f}^2 = k^2 n_\text{f}^2 - \beta^2$	$n_\text{f} = 1.5$
		thickness $d = 3.0\,\mu\text{m}$	
s	substrate	$\gamma_\text{s}^2 = \beta^2 - k^2 n_\text{s}^2$	$n_\text{s} = 1.43$
	wavelength $\lambda = 633\,\text{nm}$, $k = 2\pi/\lambda$		

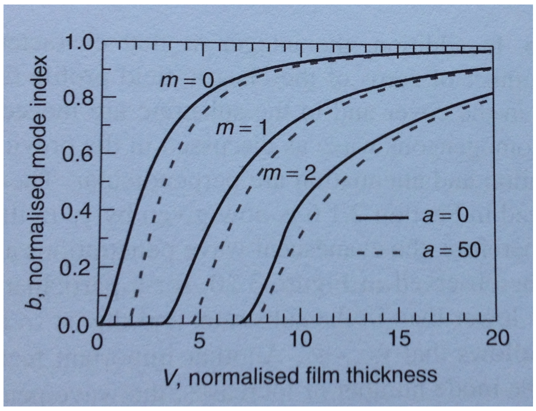


Abbildung 6.1: (*left*) Planar waveguide on a substrate, with refractive indices and thickness d. (*right*) Dispersion relation of guided modes: propagation constants β vs. normalised thickness. Guess the meaning of the label m. The parameter $a = (n_\text{s}^2 - n_\text{c}^2)/(n_\text{f}^2 - n_\text{s}^2)$ is a measure of the asymmetry (substrate and cladding differ in index). The normalised propagation constant ('mode index') is $b = ((\beta c/\omega)^2 - n_\text{s}^2)/(n_\text{f}^2 - n_\text{s}^2)$, and the normalised thickness $V = (\omega d/c)\sqrt{n_\text{f}^2 - n_\text{s}^2}$. Picture 3.17 and 3.19 taken from the book '*Integrated Photonics*' by G. Lifante (Wiley & Sons 2003).

(3) From Maxwell's equations in this planar structure, one finds that the electromagnetic fields along the x-direction satisfy an equation similar to the Schrödinger equation for a particle in a square potential. One gets for a certain polarisation (TE, electric field

mainly along the y-axis) the following transcendental equation for the mode parameter β:

$$\tan \kappa_{\rm f} d = \frac{\gamma_{\rm c}/\kappa_{\rm f} + \gamma_{\rm s}/\kappa_{\rm f}}{1 - (\gamma_{\rm c}/\kappa_{\rm f})(\gamma_{\rm s}/\kappa_{\rm f})} \tag{6.1}$$

where the parameters are given in Fig. 6.1 above. Find solutions to equation (6.1) numerically or graphically. Compare to the plot from the book by G. Lifante (*Integrated Photonics*, Wiley & Sons Inc. 2003) shown above. Make a sketch of the allowed modes for waveguides with normalised thickness $V = 2$ and $V = 10$. The choice $V = 2$ is useful because the fibre is the 'mono-mode'. For the telecom wavelength ('C band') $\lambda \simeq 1.55\,\mu\text{m}$ and the indices listed above, this corresponds to which fibre thickness?

(4) For fibre communications, the timing of optical pulses sent through the fibre is essential. Show that when a pulse is sent through a fibre of length L, it takes a travel time

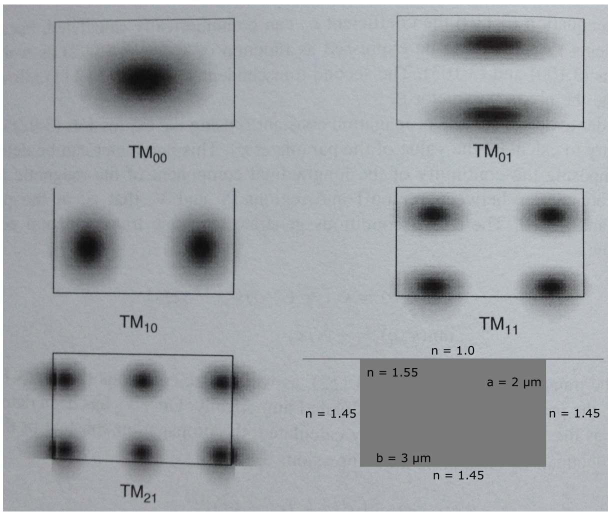
$$T = \frac{L}{v_g}, \qquad \frac{1}{v_g} = \frac{\mathrm{d}\beta}{\mathrm{d}\omega} \tag{6.2}$$

where v_g is the group velocity. Use the right plot in Fig. 6.1 to find an estimate for v_g for the mode $m = 0$. Find information about these timings on the web and justify why engineers present their results in units of $\mathrm{ps/nm}$.

(5) The data rate in modern fibers is limited by 'group velocity dispersion' (GVD). Find information about this on the web, try to understand the units for GVD, and relate this to the 'timing equation' (6.2) above.

Aufgabe 6.2 – Guided mode patterns (8 Punkte)

The figure below sketches the electric field distribution for a few modes in a rectangular waveguide (or fibre). The intensity (squared modulus of the electric field) is shown, the



wavelength $\lambda = 1.3\,\mu\mathrm{m}$

Abbildung 6.2: Mode patterns (normalised intensity in gray scale) for a rectangular guide. Bottom right: sketch of the geometry. The guide is 'buried' in a substrate. Adapted from Fig.3.35 in Lifante's book (see above).

polarisation (TM) is 'mainly vertical'.

(1) Interpret the mode labels TM_{00}, TM_{01} etc. Find a symmetry axis and comment on the parity (even, odd) of the mode functions. What features of the modes can tell you that the waveguide is 'asymmetric'? (Vacuum above the guide, embedded in a substrate to the left, right, and below, see bottom right sketch.)

(2) If you look at the intensity map carefully, you may see discontinuities at the boundaries of the waveguide. If these are physical, what could they tell you about the (local) polarisation of the mode? [5 Bonus points.]

(3) Sketch a few mode patterns with small 'labels' for a fibre with circular or elliptical cross section, using a similar notation for the 'quantum numbers'.