

# Advanced Quantum Optics

Winter term 2005/06

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**Exercise session 1**     Hand out: 20 October 2005     hand in: 27 October 2005

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## **Problem 1.1** – Heisenberg operators (5 points)

Given the standard free Hamiltonians (i.e., ignoring any interactions), compute the free time evolution of (i) the annihilation operator  $\hat{a}$  of a single field mode; (ii) the fermionic annihilator  $\sigma$  (with  $\sigma(t=0) = |g\rangle\langle e|$ ).

## **Problem 1.2** – Thermal field ‘state’ (5 points)

The  $Q$ -function of a single field mode is defined by the diagonal matrix element  $Q(\alpha) := (1/\pi)\langle\alpha|\hat{\rho}|\alpha\rangle$  of the density operator  $\hat{\rho}$  with respect to the coherent states  $|\alpha\rangle$ . The thermal ‘state’ of the mode is defined by a density operator which is a normalized sum of projectors on number states (stationary states),  $|n\rangle\langle n|$ , with Boltzmann-like weights  $\propto e^{-\beta E_n}$  determined by the energies  $E_n$  of the number states.

Compute the  $Q$ -function for the thermal state. Make a sketch and discuss its width as a function of temperature.